Generalized Brans-Dicke cosmology in the presence of matter and dark energy

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Abstract

We study the Generalized Brans-Dicke cosmology in the presence of matter and dark energy. Of particular interest for a constant Brans-Dicke parameter, the de Sitter space has also been investigated.

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1. INTRODUCTION

The Brans-Dicke (BD) theory is defined by a scalar field φ and a constant coupling function ω as perhaps the most natural extension of general theory of relativity that is obtained in the limit of $\omega \to \infty$ and $\varphi =$ constant. The theory appears naturally in supergravity theory, Kaluza-Klein theories and in all the known effective string actions. In FRW cosmology, the theory gives simple expanding solutions for scale factor $a(t)$ and scalar field $\varphi(t)$ which are compatible with solar system experiments [1]

The generalized BD theory, sometimes referred to as graviton-dilaton or scalar -tensor theory, is instead, defined by ω which is implicitly a function of time $\omega(t)$. Naturally, a few attempts have been taken to study the dynamics of the universe using this formalism [2][3][4].

The belief that modified gravity theories may have played a crucial role during the early universe has recently been renewed by extended inflation (for example see [5][6]). In this case a scalar tensor gravity theory allows the first order phase transition of the old inflationary model to complete. This arises because the scalar field φ, that is essentially the inverse of the Newtons gravitational constant, damps the rate of expansion and, in the original extended inflationary model based on the BD theory, turns the exponential expansion found in general relativity into power law inflation [7]. However, BD theory is unable to meet the simultaneous and distinct requirements placed by the postNewtonian solar system tests and by the need to keep the sizes of the bubbles nucleated during inflation within the limits permitted by the anisotropies of the microwave background [8] [9].

In order to carry out a detailed study of the dynamics of the cosmic evolution in this formalism, knowledge about exact time-dependence of $a(t)$, $\varphi(t)$ and $\omega(t)$ and energy momentum distribution in spacetime is essential. In part of this paper, similar to [10], we have obtained red shift-dependence of $\omega(z)$ with the power of red shift z determined in terms of the exponent of scale factor $a(t)$ which is taken to vary as $a(t) \propto t^{\delta}$ and equation of state parameter for the matter and dark energy contribution. With the help of observational evidence we obtain certain information about the parameters describing the cosmological model in particular regarding the early and late time behavior of the universe. We also investigate both empty and filled de Sitter space case with constant BD parameter and shows that the result is consistent with recent measurements.

2. THE MODEL

We consider a flat Universe filled with pressureless matter and dark energy both described by perfect fluid. The field equations in generalized BD theory with time dependent ω, are

$$H^2\varphi^2 + H\dot{\varphi}\varphi - \frac{\omega}{6}\dot{\varphi}^2 = \frac{\rho_x + \rho_m}{3}\varphi, \tag{1}$$

$$2\dot{H}\varphi^2 + 3H^2\varphi^2 + \frac{\omega}{2}\dot{\varphi}^2 + 2H\dot{\varphi}\varphi + \ddot{\varphi}\varphi = -p_x\varphi, \tag{2}$$

where $p_x = \alpha_x\rho_x$, $p_m = \alpha_m\rho_m$ are the equations of state for dark energy and matter and the scale factor and scalar field are respectively $a(t)$ and $\varphi(t)$. In addition, the equation of motion for BD scalar field is given by

$$\ddot{\varphi} + 3H\dot{\varphi} = \frac{\rho_x + \rho_m - 3p_x}{2\omega + 3} - \frac{\dot{\omega}\dot{\varphi}}{2\omega + 3}\cdot \tag{3}$$

From equations (1), (2) and (3), the energy conservation equation can be obtained as

$$(\dot{\rho}_x + \dot{\rho}_m) + 3H(\rho_x + \rho_m + p_x) = 0\cdot \tag{4}$$

Note that the wave equation for the BD scalar field, (3), is not an independent expression as it follows from the Bianchi identities alongside equations (1) and (2). In addition, the dynamics of the scale factor is governed not only by the matter and dark energy, but also by the BD scalar field, $\varphi(t)$.

One may assume that the matter and dark energy interact with each other, thus the growth of one is at the expense of the other. Then the conservation equations for them are

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{5}$$

$$\dot{\rho}_x + 3H(1+\alpha_x)\rho_x = -Q, \tag{6}$$

where $Q > 0$ stands for the interaction term. Alternately, one could construct the equivalent uncoupled model described by:

$$\dot{\rho}_m + 3H(1 - \alpha_{m,eff})\rho_m \;=\; 0, \tag{7}$$

$$\dot{\rho}_x + 3H(1 + \alpha_{x,eff})\rho_x \;=\; 0, \tag{8}$$

where

$$\alpha_{m,eff} = \frac{Q}{3H\rho_m}, \tag{9}$$

$$\alpha_{x,eff} = \alpha_x + \frac{Q}{3H\rho_x}\cdot \tag{10}$$

The wave equation (3) is not altered by the interaction equations (5) and (6), since although the matter and dark energy components do not conserve separately the overall fluid -matter plus dark energy- does. Thus, one may introduce the total energy density $\rho_{tot} = \rho_m + \rho_x$, and from equation (7) and (8), obtains

$$\dot{\rho}_{tot} + 3H(1+\alpha_{tot})\rho_{tot} \;=\; 0, \tag{11}$$

with the solution

$$\rho_{tot} \propto a^{-3(1+\alpha_{tot})}, \tag{12}$$

where

$$\alpha_{tot} = \frac{p_x}{\rho_m + \rho_x} = \alpha_x \Omega_x, \tag{13}$$

and $\Omega_x \equiv \frac{\rho_x}{\rho_{tot}}$. One can also find the rate of Ω_x as

$$\dot{\Omega}_x = \frac{-3H(\alpha_{x,eff} + \alpha_{m,eff})\rho_x \rho_m}{\rho_{tot}^2} = -\frac{3H\alpha_{tot}\rho_m + Q}{\rho_{tot}}, \tag{14}$$

or in terms of red shift z,

$$\Omega_x' = \frac{3(\alpha_{x,eff} + \alpha_{m,eff})\rho_x \rho_m}{\rho_{tot}^2(1+z)}, \tag{15}$$

where $\frac{1}{a} = 1+z$, and $a = 1$ is the present value of the scale factor and "$'$" means derivative with respect to z.

Also in terms of the red shift z, equation (8) can be rewritten as

$$\rho_x' = \frac{3(1+\alpha_{x,eff})\rho_x}{1+z}, \tag{16}$$

where the equation based on the sign of $1+\alpha_{x,eff}$ shows whether the density of dark energy will increase or not as the red shift becomes low. For positive sign, the density decreases like the quintessence, for negative sign, it increases like the phantom, and when it is zero the density is invariant like the cosmological constant.

3. THE GENERAL ω

Following paper [2] for $a \propto t^\delta$, $\varphi \propto t^\beta$ and time dependent ω, equation (1) gives,

$$\omega(t) = -\frac{2}{\beta^2} t^{-3\delta(1+\alpha_{tot})-\beta+2}. \tag{17}$$

One can rewrite equation (17) in terms of red shift z,

$$\omega(z) = -\frac{2}{\beta^2}(1+z)^{3(1+\alpha_{tot})+(\beta-2)/\delta}, \tag{18}$$

where its derivative with respect to z is given by

$$\omega' = \frac{2(-3\delta(1+\alpha_{tot})-\beta+2)}{\beta^2\delta}(1+z)^{2+3\alpha_{tot}+(\beta-2)/\delta}. \tag{19}$$

After some calculations, we can also rewrite equations (1) and (3) as

$$[(\frac{\dot{a}}{a}+\frac{\dot{\varphi}}{2\varphi})^2 - \frac{(2\omega+3)\dot{\varphi}^2}{12\varphi^2}]3\varphi = \rho_{tot}, \tag{20}$$

$$\beta[\frac{3(1-\alpha_{tot})}{4}\beta + \frac{3\delta(1-\alpha_{tot})}{2}] = 0. \tag{21}$$

From equation (21) one finds that for $\alpha_{tot} \neq 1$, β restricted to be 0 or -2δ. In case of $\alpha_{tot} = 1$, there is no constraint on β. For $\beta = 0$, from equation (18) one finds that $\omega \to \infty$ and from equation (20) we obtain $\varphi = \varphi_0 = constant$ and $a \propto t^\delta$. So for $\beta = 0$, Brans-Dicke model goes over to General Relativity [11] and to obtain δ, one has to solve equations of General Relativity [12]. In case of $\beta = -2\delta$, equations (18) and (19) reduce to,

$$\omega(z) = -\frac{1}{2\delta^2}(1+z)^{(1+3\alpha_{tot})-2/\delta}, \tag{22}$$

$$\omega' = \frac{-\delta(1+3\alpha_{tot})+2}{2\delta^3}(1+z)^{3\alpha_{tot}-2/\delta}. \tag{23}$$

It is clear from equations (22) that, the parameter α_{tot} which takes different values in different era, controls the z dependence of ω in different era. For today value of $z = 0$, we have

$$\omega_0 = -\frac{1}{2\delta^2}, \tag{24}$$

and

$$\omega_0' = \frac{-\delta(1+3\alpha_{tot})+2}{2\delta^3}, \tag{25}$$

where for the present acceleration of the universe that δ needs to be greater than one, ω_0 given by (24) has the minimum value of 1/2 in agreement with the observation [13].

Further, in the last scattering surface, during the galaxy formation era ($1 < z < 3$) where dark energy density must be sub-dominant to matter density ($\alpha_{tot} > -0.5$), we have $-1/12 < \omega < -1/14$.

In the Big Bang Nucleosynthesis (BBN) era where the presence of dark energy should not disturb the observed Helium abundance in the universe ($(\alpha_{tot})_{BBN} > -0.21$ at $z = 10^{10}$)

[14], we have $-9/32 < \omega < -9/128$. This also shows that at sometimes in the future, $z = -1$, we have a big rip and $\omega \rightarrow -\infty$.

One also finds from (22) and (23) that

$$\frac{\omega'}{\omega} = ((1 + 3\alpha_{tot}) - 2/\delta)(1 + z)^{-1}, \tag{26}$$

where the ration for today is negative, in the distance future for $z = -1$ it goes to minus infinity and in the distance past where $z \rightarrow \infty$, it approaches zero.

4. DE SITTER SPACE TIME WITH CONSTANT ω

We now assume an empty de Sitter spacetime with the solution $H = H_0$. Then, for $\varphi \propto e^{\beta}$ and constant BD parameter ω, equations (1) and (2) can be solved:

$$H_0^2 + H_0\beta - \frac{\omega}{6}\beta^2 = 0, \tag{27}$$

$$3H_0^2 + 2H_0\beta + (\frac{\omega}{2} + 1)\beta^2 = 0, \tag{28}$$

to give

$$\beta = \frac{(-2 \pm \sqrt{-8 - 6\omega})H_0}{2 + \omega}, \tag{29}$$

$$\beta = \frac{(3 \pm \sqrt{9 + 6\omega})H_0}{\omega}. \tag{30}$$

For these two solutions to be consistent implies that $\omega = -4/3$ or $\omega = -3/2$. For equations (1), (2) and (3) to be simultaneously satisfied only $\omega = -4/3$ and so $\beta = -3H_0$ is acceptable. For large value of H_0, during inflation, while the universe expands exponentially, the BD scalar field drops exponentially.

In the presence of matter and dark energy we may also have a de Sitter solution $H = H_0$. Then the equations (1) and (2) become

$$H_0^2 + H_0\beta - \frac{\omega}{6}\beta^2 = \frac{1}{3}e^{[-3H_0(1+\alpha_{tot})-\beta]t}, \tag{31}$$

$$3H_0^2 + 2H_0\beta + (\frac{\omega}{2} + 1)\beta^2 = -\alpha_{tot}e^{[-3H_0(1+\alpha_{tot})-\beta]t}. \tag{32}$$

These equations are satisfied when

$$H_0^2 + H_0\beta - \frac{\omega}{6}\beta^2 = \frac{1}{3}, \tag{33}$$

$$3H_0^2 + 2H_0\beta + (\frac{\omega}{2} + 1)\beta^2 = -\alpha_{tot}, \tag{34}$$

$$-3H_0(1 + \alpha_{tot}) = \beta. \tag{35}$$

Using equation (35) in (33) one gets

$$\omega = \frac{2(-3H_0^2(2+3\alpha_{tot})-1)}{9H_0^2(1+\alpha_{tot})^2}. \tag{36}$$

Similarly, equation (35) in conjunction with equation (34) gives

$$\omega = \frac{2(3H_0^2(1+2\alpha_{tot})-9H_0^2(1+\alpha_{tot})^2-\alpha_{tot})}{9H_0^2(1+\alpha_{tot})^2}. \tag{37}$$

For equations (33), (34) and (35) to be simultaneously satisfied, above two values of ω should be equal. Imposition of this condition leads to

$$\alpha_{tot} = \frac{-(3H_0^2+1)\pm\sqrt{9H_0^4+42H_0^2+1}}{18H_0^2}. \tag{38}$$

From the above solution we find that for negative sign and $H_0 > 10$, $\alpha_{tot} = -0.33$ or for $H_0 < 0.15$, $\alpha_{tot} > -0.33$. For positive sign, for $H_0 \gg 1$ or $H_0 \simeq 0$, we have $\alpha_{tot} \simeq 0$. In case of $\alpha_{tot} > -0.33$ or from equation (13), $\Omega_x < 0.33$, this is consistent with last Scattering Surface, during the galaxy formation era ($1 < z < 3$) where dark energy density must be sub-dominant to matter density and accordingly $\Omega_x < 0.5$. Then one gets $\omega = -1.5$ and $\beta = -142.7$.

From the above argument as to the Brans-Dicke parameters ω are concerned, it is negative and of the order of unity. This could be considered as an unsatisfactory result, in view of the high lower limits imposed to ω by astronomical tests in the Solar System.

A possible solution of this contradiction as discussed is in considering a non-constant coupling function $\omega(t)$ in Generalised Brans-Dicke theory. Thus, the value of such a function can change with the cosmic time and, in the limit $t \to \infty$, it could agree with local measured values [15]. This argument is based on the scalar-tensor theories in which ω depends on the scale, being very high in the weak field approximation of Solar System that probe only a limited range of space and time. To effectively constrain more general scalar-tensor theories, one would also like to have strong-field experiments, such as that provided by the binary pulsar [16]. It was also pointed out that in cosmological models based on more general scalar-tensor theories in which ω can vary, there is generally an attractor mechanism that drives ω to ∞ at late times [17].

5. CONCLUSION

In this work,we have derived the explicit time and red shift dependence of the Brans-Dicke parameter ω by solving gravitational field and wave equations of generalized BD theory consistently in the present of matter and dark energy, assuming power law behavior for the scale factor $a(t)$ and scalar field $\varphi(t)$. Similar to the work done in [2], we find two consistent solutions of the field and wave equations. One solution leads to General Relativity and the other one leads to a z-dependent $\omega(z)$ whose red shift dependence is governed by the equation of state parameter α_{tot}. Consequently, $\omega(z)$ exhibits different behavior in different epochs of the evolving Universe characterized by its dominant matter/dark energy components. We also find that the ratio $\frac{\omega'}{\omega}$ is a negative monotonically increasing function of z. In particular, for an expanding universe, we have studied the empty de Sitter space with constant ω and find that in the era of inflation that H_0 is high, the scalar field drops exponentially and $\omega = -4/3$. Moreover, in the presence of matter and dark energy, we also find that ω is negative and we are able to explain the last scattering surface constraint

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